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Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Thursday 16 May 2019

Afternoon | Paper Reference **8FM0-26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J only)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\ \text{m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ▶

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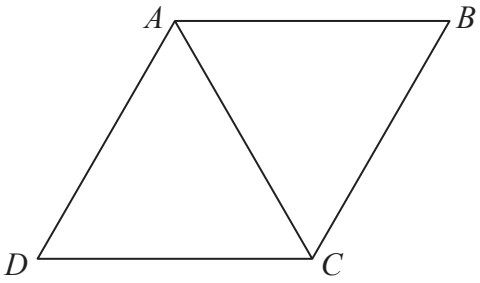

Figure 1

Five identical uniform rods are joined together to form the rigid framework $ABCD$ shown in Figure 1. Each rod has weight W and length $4a$. The points A, B, C and D all lie in the same plane.

The centre of mass of the framework is at the point G.

(a) Explain why G is the midpoint of AC.

(1)

The framework is suspended from the ceiling by two vertical light inextensible strings. One string is attached to the framework at A and the other string is attached to the framework at B. The framework hangs freely in equilibrium with AB horizontal.

(b) Find

(i) the tension in the string attached at A,

(ii) the tension in the string attached at B.

(4)

A particle of weight kW is now attached to the framework at D and a particle of weight $2kW$ is now attached to the framework at C. The framework remains in equilibrium with AB horizontal and the strings vertical.

Either string will break if the tension in it exceeds $6W$.

(c) Find the greatest possible value of k.

(4)

a) Since the rods are uniform, and the axes of symmetry (which are the perpendicular bisector of AC and the line through AC) intersect at midpoint of AC which is G

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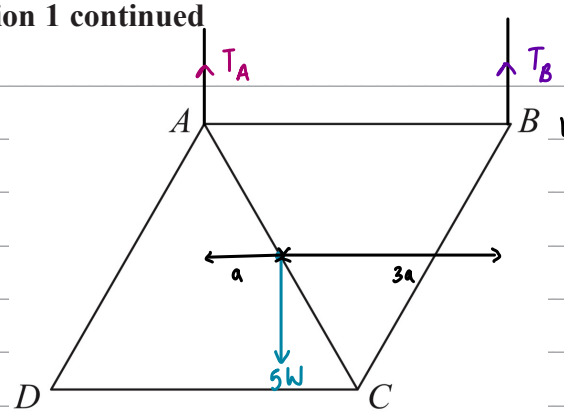
Question 1 continued

b)



i) By taking moments about B:

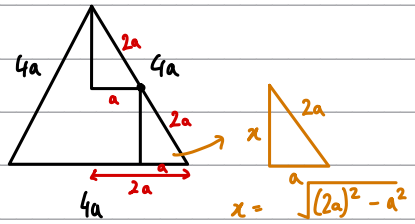
$3a\,(5W) = 4a\,(T_A)$

$T_A = \frac{15W}{4}$

ii) By taking moments about A:

$a\,(5W) = 4a\,(T_B)$

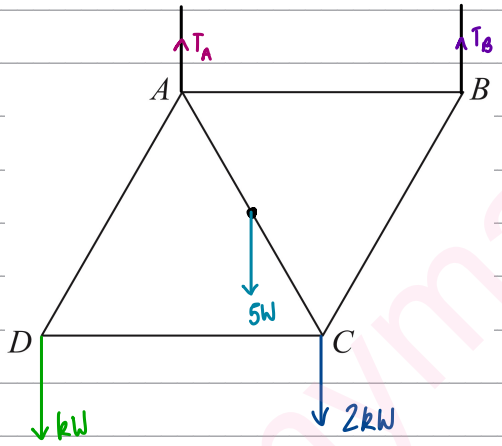
$T_B = \frac{5}{4}W$



$x = \sqrt{(2a)^2 - a^2}$

$x = a\sqrt{3}$

c)



By taking moments about A:

$4a \times T_B + 2a \times kW = a \times 5W + 2a \times 2kW$

$4aT_B + 2akW = 5aW + 4akW$

$4T_B = 5W + 4kW$

$T_B = \frac{W(5+2k)}{4}$

By taking moments about B:

$4a \times T_A = 5W \times 3a + 2kW \times 2a + kW \times 6a$

$4T_A = 15W + 4kW + 6kW$

$T_A = \frac{W(15+10k)}{4}$

$\frac{W(15+10k)}{4} > \frac{W(5+2k)}{4}$ so $T_A > T_B$

(when comparing numerators where k is a positive value since weight can't be negative)

T_A will be larger so when $T_A = 6W$

$\therefore \quad 6W = \frac{W(15+10k)}{4}$

$24 = 15 + 10k$

$9 = 10k \Rightarrow k = \frac{9}{10} = 0.9$

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Question 1 continued

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Question 1 continued

(Total for Question 1 is 9 marks)

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2. A car moves in a straight line along a horizontal road. The car is modelled as a particle. At time t seconds, where $t \geqslant 0$, the speed of the car is v m s^{-1}

At the instant when $t = 0$, the car passes through the point A with speed 2 m s^{-1}

The acceleration, a m s^{-2}, of the car is modelled by

$$a = \frac{4}{2+v}$$

in the direction of motion of the car.

(a) Use algebraic integration to show that $v = \sqrt{8t+16} - 2$

(6)

At the instant when the car passes through the point B, the speed of the car is 4 m s^{-1}

(b) Use algebraic integration to find the distance AB.

(6)

a) $a = \frac{dv}{dt}$

$\frac{dv}{dt} = \frac{4}{2+v}$

$\int (2+v)\,dv = \int 4\,dt$

$2v + \frac{v^2}{2} = 4t + c$

Using boundary conditions

$t = 0,\ v = 2$ $\quad 2(2) + \frac{2^2}{2} = 4(0) + c$

$c = 6$

$\therefore 2v + \frac{v^2}{2} = 4t + 6$

$v^2 + 4v = 8t + 12$

$(v+2)^2 - 4 = 8t + 12$

$(v+2)^2 = 8t + 16$

$v + 2 = \sqrt{8t+16}$

$v = \sqrt{8t+16} - 2$

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Question 2 continued

b) $v = \frac{dx}{dt}$

$$\frac{dx}{dt} = \sqrt{8t+16} - 2$$

Separating variables and integrate

$$\int \sqrt{8t+16} - 2 \, dt = \int dx$$

$$2^{\frac{3}{2}} \int \sqrt{t+2} \, dt - \int 2 \, dt = x$$

Using integration by substitution

Let $u = t+2$

$$\int \sqrt{u} \, du = \frac{2u^{\frac{3}{2}}}{3} = \frac{2(t+2)^{\frac{3}{2}}}{3}$$

$$2^{\frac{3}{2}} \left(\frac{2(t+2)^{\frac{3}{2}}}{3} \right) - 2t = x + C$$

$$\frac{2^{\frac{5}{2}}(t+2)^{\frac{3}{2}}}{3} - 2t = x + C$$

Using boundary conditions

$x = 0, t = 0$

$$\frac{2^{\frac{5}{2}}(0+2)^{\frac{3}{2}}}{3} - 2(0) = C$$

$$C = \frac{16}{3}$$

$$\therefore \quad x = \frac{2^{\frac{5}{2}}(t+2)^{\frac{3}{2}}}{3} - 2t - \frac{16}{3}$$

At $v = 4$, $4 = \sqrt{8t+16} - 2$

$6 = \sqrt{8t+16}$

$36 = 8t + 16$

$20 = 8t$

$t = \frac{10}{4}$

At $t = \frac{10}{4}$, $x = \frac{2^{\frac{5}{2}}\left(\frac{10}{4}+2\right)^{\frac{3}{2}}}{3} - 2\left(\frac{10}{4}\right) - \frac{16}{3}$

$x = 18 - 5 - \frac{16}{3}$

$x = \frac{23}{3}$ m

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Question 2 continued

(Total for Question 2 is 12 marks)

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3. A light inextensible string has length $8a$. One end of the string is attached to a fixed point A and the other end of the string is attached to a fixed point B, with A vertically above B and $AB = 4a$. A small ball of mass m is attached to a point P on the string, where $AP = 5a$.

The ball moves in a horizontal circle with constant speed v, with both AP and BP taut.

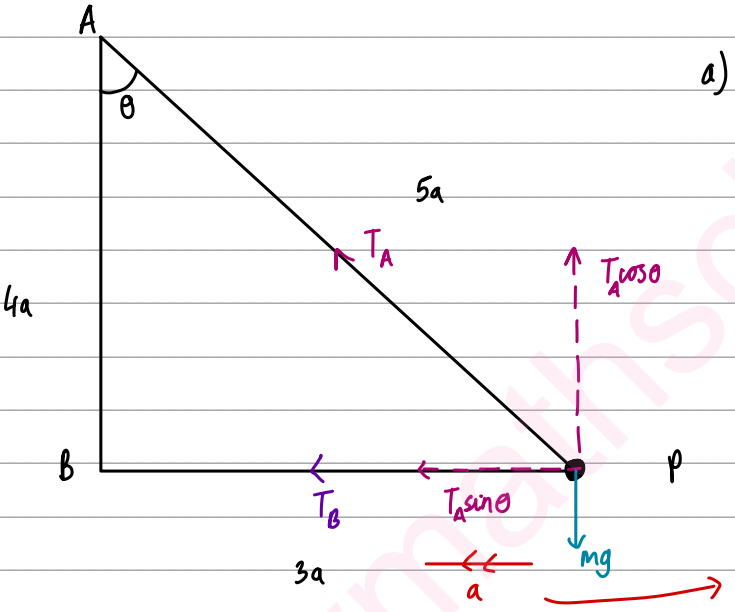
The string will break if the tension in it exceeds $\frac{3mg}{2}$

By modelling the ball as a particle and assuming the string does not break,

(a) show that $\frac{9ag}{4} < v^2 \leqslant \frac{27ag}{4}$ **(7)**

(b) find the least possible time needed for the ball to make one complete revolution. **(2)**

a)



a) Using trigonometry

$\cos\theta = \frac{4}{5}$

$\sin\theta = \frac{3}{5}$

$a = \frac{v^2}{r}$ where v is angular speed and r is radius of circle

Resolving vertically (↑)

$T_A\cos\theta = mg$

$\frac{4}{5}T_A = mg$

$T_A = \frac{5}{4}mg$

Resolving horizontally (←)

$T_B + T_A\sin\theta = \frac{mv^2}{3a}$

$T_B = \frac{mv^2}{3a} - \frac{3}{4}mg$

$T_B > 0$ since BP is taut, $0 < \frac{mv^2}{3a} - \frac{3}{4}mg$

$\frac{3}{4}mg < \frac{mv^2}{3a}$

$\frac{9ag}{4} < v^2$

$T_B \leqslant \frac{3mg}{2}$ since string does not break, $\frac{mv^2}{3a} - \frac{3}{4}mg \leqslant \frac{3mg}{2}$

$\frac{mv^2}{3a} \leqslant \frac{9mg}{4} \Rightarrow v^2 \leqslant \frac{27ag}{4}$

$\Rightarrow \frac{9ag}{4} < v^2 \leqslant \frac{27ag}{4}$

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Question 3 continued

b) The max velocity : $v^2 = \frac{27ag}{4}$ from part b)

$$T = \frac{d}{s} = \frac{2\pi r}{v} = \frac{2\pi(3a)}{\sqrt{\frac{27ag}{4}}}$$

$$T\sqrt{\frac{27ag}{4}} = 6\pi a$$

$$T^2\left(\frac{27ag}{4}\right) = 36\pi^2a^2$$

$$T^2 = 36\pi^2a^2 \times \frac{4}{27ag}$$

$$T^2 = \frac{144\pi^2a^2}{27ag}$$

$$T = \sqrt{\frac{144\pi^2a^2}{27ag}} = \frac{12\pi\sqrt{a}}{3\sqrt{3g}} = 4\pi\sqrt{\frac{a}{3g}}$$

$$T = 4\pi\sqrt{\frac{a}{3g}}$$

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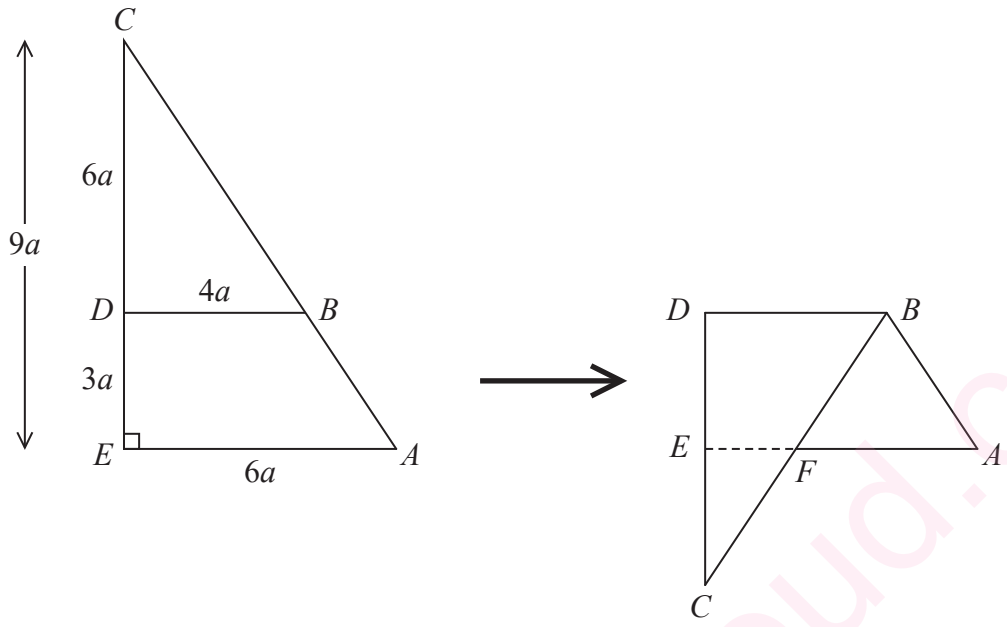

Triangular lamina Folded lamina

Figure 2

The uniform triangular lamina $ABCDE$ is such that angle $CEA = 90°$, $CE = 9a$ and $EA = 6a$. The point D lies on CE, with $DE = 3a$. The point B on CA is such that DB is parallel to EA and $DB = 4a$. The triangular lamina is folded along the line DB to form the folded lamina $ABDECF$, as shown in Figure 2.

The distance of the centre of mass of the triangular lamina from DC is d_1

The distance of the centre of mass of the folded lamina from DC is d_2

(a) Explain why $d_1 = d_2$

(1)

The folded lamina is freely suspended from B and hangs in equilibrium with BA inclined at an angle α to the downward vertical through B.

(b) Find, to the nearest degree, the size of angle α.

(9)

a) In the process of folding, each point of the lamina remains the same distance from CD. The fold occurs in the x direction/axis so it doesn't affect how far the COM lies in the x direction, only y direction as these are the points/coordinates that change from the fold.

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Question 4 continued

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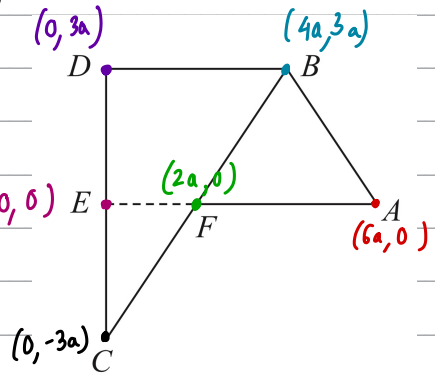


Splitting the lamina into 3 shapes

	Large triangle (ACE)	Removed triangle (BCD)	Added triangle (BCD)	Folded lamina
Area	$27a^2$	$12a^2$	$12a^2$	$27a^2$

COM $\bar{y}$ $\frac{9a+0+0}{3} = 3a$ $\frac{3a+3a+9a}{3} = 5a$ $\frac{3a+3a-3a}{3} = a$ $\bar{y}$

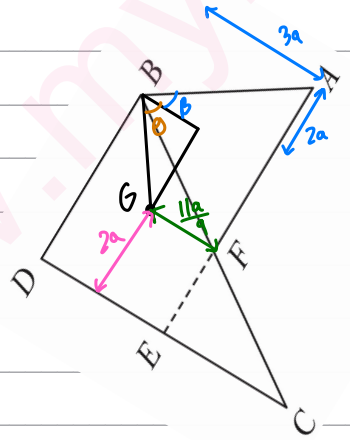
moments = force x perpendicular distance
The sum of moments is equal to the overall moment acting through the COM.
Mathematically → $\sum m_i x_i = \bar{x} \sum m_i$
Where m = force
x = perpendicular distance

Using part a), $\bar{x} = 2a = d_2$ since COM $\bar{x}$ for unfolded lamina = $\frac{0+0+6a}{3} = 2a$

Replacing force with area as they are proportional

$27a^2(3a) - 12(5a) + 12(a) = 27\bar{y}$

$\bar{y} = \frac{11a}{9}$



$\alpha = \theta + \beta$

$\tan\theta = \frac{4a-\bar{x}}{3a-\bar{y}} = \frac{2a}{\frac{16a}{9}}$

$\tan\theta = \frac{9}{8}$

$\theta = \arctan\left(\frac{9}{8}\right)$

$\tan\beta = \frac{2a}{3a} = \frac{2}{3}$

$\beta = \arctan\left(\frac{2}{3}\right)$

$\alpha = \arctan\left(\frac{9}{8}\right) + \arctan\left(\frac{2}{3}\right)$

$\alpha = 82.1°$ (3sf)

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Question 4 continued

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Question 4 continued

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TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS

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